M.Sc. 1st Semester Examination-2022-23

PHYSICS

Course ID : 12451 **Course Code : PHYS/101C**

Course Title : Mathematical Methods–I & Classical Mechanics

Time : 2 Hours *Full Marks* : 40

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Unit–I

1. Answer any *three* of the following : 2×3=6

(a) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in Laurent series valid for $1 < |z| < 3$.

(b) Determine the poles and the residue at simple pole of the function $f(z)=\frac{z^2}{(z-1)^2(z+2)}$.

(c) A function u(x, y) + i v(x, y) is analytic, find v(x, y), where u(x, y) = exp(x) (xcosy − ysiny).

(d) Explain the idea of branch singularity and branch cut.

(e) Show that the eigen values of a Hermition matrix are real.

2. Answer any *two* of the following : 4×2=8

(a) Show that Tr(AB) = Tr(BA).

(b) Show that the vectors (3i, 1, 0), (2, −i, 1) and (0, 1 + 1, 1 − i) form a basis of the complex vector space.

(c) Evaluate $\int_0^\infty \frac{\sin ax}{x} dx$, where a > 0.

(d) Show that the real and imaginary parts of the function w = log z satisfy and Cauchy-Riemann equation when z is not zero. Find its derivative.

3. Answer any *one* of the following : 6×1=6

(a) If f(z) = u + iv is an analytic function of z and

$$u - v = \frac{\cos z + \sin x - e^{-y}}{2\cos x - 2\cosh y}, \text{ prove that } f(z) = \frac{1}{2}\left[1 - \cot\frac{\pi}{2}\right]$$

when $f\left(\frac{\pi}{2}\right) = 0$.

(b) Define vector space. Write all axioms to be satisfied to defind a vector space. 2+4

Unit–II

4. Answer any *three* of the following : 2×3=6

(a) Show that if the Hamiltonian of a system does not contain a particular coordinate, the corresponding momentum is conserved.

(b) Show the Poisson bracket of a conserved quantity with Hamiltonian is zero.

(c) Show that if a given coordinate is cyclic in the Lagrangian, it will also be cyclic in Hamiltonian.

(d) Show that the generalized momentum corresponding to a cyclic coordinate is constant of motion.

(e) What is Noether theorem ?

5. Answer any *two* of the following : 4×2=8

(a) State and prove Jacobi-Poisson's theorem.

(b) Show that the transformation $P = \frac{1}{2}\left(p^2 + q^2\right)$,

$Q = \tan^{-1}(q/p)$ is cononical.

(c) Prove by the Hamolton Jacobi theory that the orbit of a planet round the sun is an elliptic one with sun at one of the foci.

(d) Solve the Hamilton-Jacobi equation for the system whose Hamiltonian is given by $H = \frac{p^2}{2} - \frac{\mu}{q}$.

6. Answer any *one* of the following : 6×1=6

(a) Given the generating function

$$F_1(q, Q, t) = \frac{1}{2} m\omega \left[q - \frac{f(t)}{m\omega^2} \right]^2 \cot Q,$$

find the transformations equations and hence obtain the equation of motion of a simple harmonic oscillator acted upon by a force f(t) in terms of Q and P.

(b) (i) Find the Poisson bracket of $[L_x, L_y]$.

(ii) Show that Poisson bracket is invariant under cononical transformation. 2+4
